SPEAR Math Sample Pack

SPEAR

The Primary Math Problem Solving Framework

A comprehensive framework that:

- supports teaching and learning in Math problem solving
- is easy to use
- is flexible
- promotes independence
- encourages the transfer of process skills
- includes a resource of over 550 problems
- can be used alongside curriculum materials
- is well-established in schools across the UK and has now been rewritten for US elementary schools

The Challenge

- Problem solving in Math is under-developed in many elementary schools
- Few elementary school teachers are Math specialists
- Math problem solving is complex and pupils need to be taught the skills required
- Many teachers feel they should do more problem solving in Math but don't know where to start
- Pupils often struggle to transfer knowledge and understanding to unfamiliar contexts

The Framework

- A five step process which children can understand, remember and use
- Supported by a comprehensive range of materials: graded problems, records, self evaluation sheets, etc.
- Can be used alongside a range of Math curricula
- Accessible to children of all ages and abilities from Preschool to Grade 6 and above
- An ideal resource to use alongside schemes such as Singapore Math

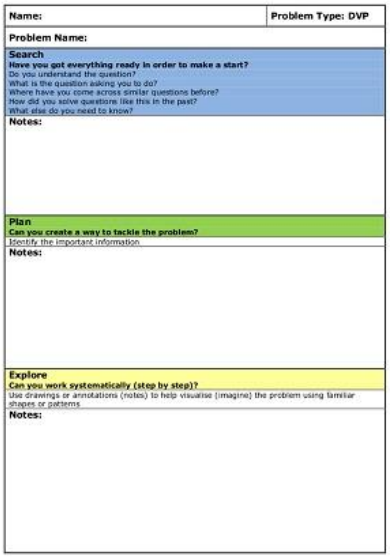
Sample Pack

The contents of this sample pack have been selected to give you a clear understanding of what you get when you purchase a license for using SPEAR Math.

Overview of SPEAR Math Contents

Activity Records

Activity record sheets for each type of problem as well as general and simplified record sheets.



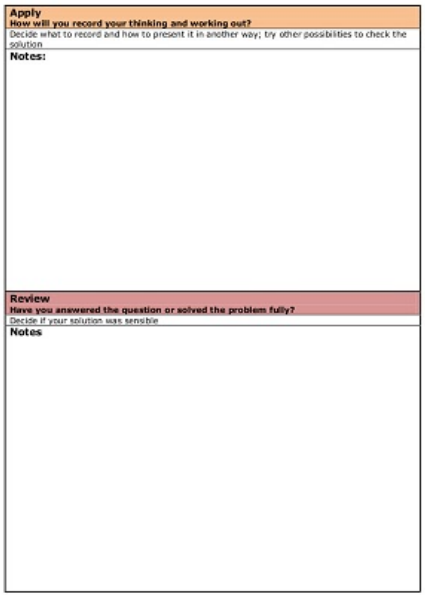
Name: | Problem Type: DVP

Problem Name:

Search
Have you got everything ready in order to make a start?
Do you understand the question?
What is the question asking you to do?
Where have you come across similar questions before?
How did you solve questions like this in the past?
What else do you need to know?
Notes:

Plan
Can you create a way to tackle the problem?
Identify the important information
Notes:

Explore
Can you work systematically (step by step)?
Use drawings or annotations (notes) to help visualise (imagine) the problem using familiar shapes or patterns
Notes:

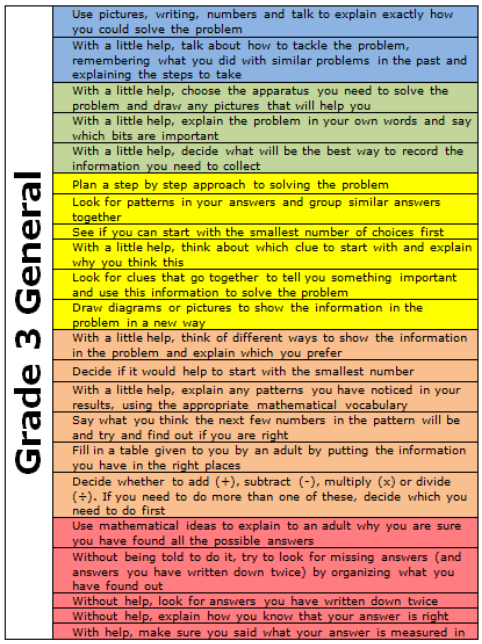


Apply
How will you record your thinking and working out?
Decide what to record and how to present it in another way; try other possibilities to check the solution
Notes:

Review
Have you answered the question or solved the problem fully?
Decide if your solution was sensible
Notes

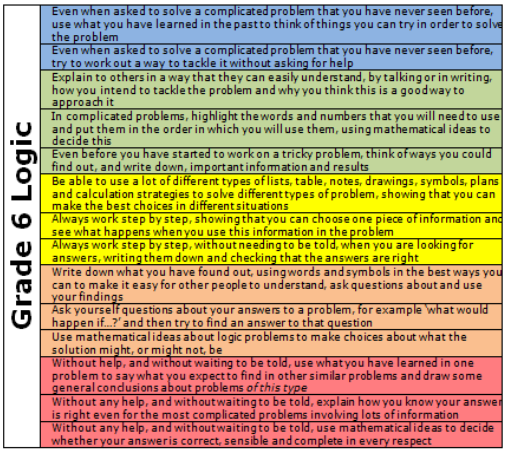
Help Me Cards

Help Me Cards for each stage of each type of problem for each grade to promote pupil independence:



Grade 3 General

- Use pictures, writing, numbers and talk to explain exactly how you could solve the problem
- With a little help, talk about how to tackle the problem, remembering what you did with similar problems in the past and explaining the steps to take
- With a little help, choose the apparatus you need to solve the problem and draw any pictures that will help you
- With a little help, explain the problem in your own words and say which bits are important
- With a little help, decide what will be the best way to record the information you need to collect
- Plan a step by step approach to solving the problem
- Look for patterns in your answers and group similar answers together
- See if you can start with the smallest number of choices first
- With a little help, think about which clue to start with and explain why you think this
- Look for clues that go together to tell you something important and use this information to solve the problem
- Draw diagrams or pictures to show the information in the problem in a new way
- With a little help, think of different ways to show the information in the problem and explain which you prefer
- Decide if it would help to start with the smallest number
- With a little help, explain any patterns you have noticed in your results, using the appropriate mathematical vocabulary
- Say what you think the next few numbers in the pattern will be and try and find out if you are right
- Fill in a table given to you by an adult by putting the information you have in the right places
- Decide whether to add (+), subtract (-), multiply (x) or divide (÷). If you need to do more than one of these, decide which you need to do first
- Use mathematical ideas to explain to an adult why you are sure you have found all the possible answers
- Without being told to do it, try to look for missing answers (and answers you have written down twice) by organizing what you have found out
- Without help, look for answers you have written down twice
- Without help, explain how you know that your answer is right
- With help, make sure you said what your answer is measured in



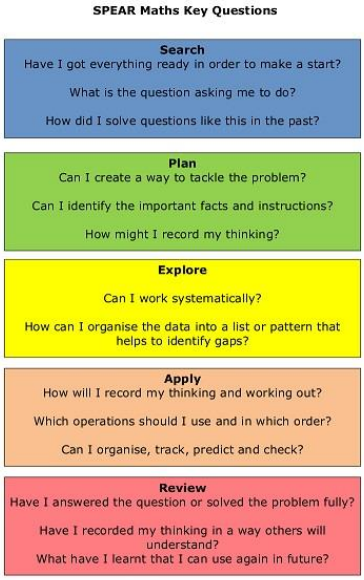
Grade 6 Logic

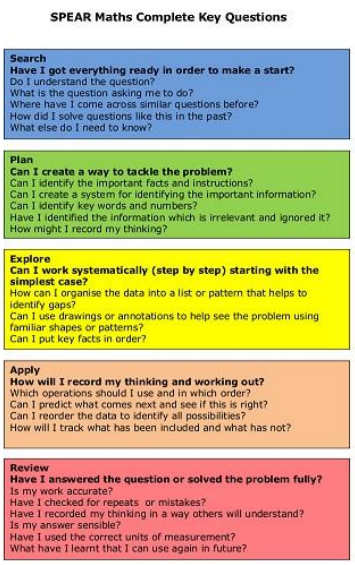
- Even when asked to solve a complicated problem that you have never seen before, use what you have learned in the past to think of things you can try in order to solve the problem
- Even when asked to solve a complicated problem that you have never seen before, try to work out a way to tackle it without asking for help
- Explain to others in a way that they can easily understand, by talking or in writing, how you intend to tackle the problem and why you think this is a good way to approach it
- In complicated problems, highlight the words and numbers that you will need to use and put them in the order in which you will use them, using mathematical ideas to decide this
- Even before you have started to work on a tricky problem, think of ways you could find out, and write down, important information and results
- Be able to use a lot of different types of lists, table, notes, drawings, symbols, plans and calculation strategies to solve different types of problem, showing that you can make the best choices in different situations
- Always work step by step, showing that you can choose one piece of information and see what happens when you use this information in the problem
- Always work step by step, without needing to be told, when you are looking for answers, writing them down and checking that the answers are right
- Write down what you have found out, using words and symbols in the best ways you can to make it easy for other people to understand, ask questions about and use your findings
- Ask yourself questions about your answers to a problem, for example 'what would happen if...?' and then try to find an answer to that question
- Use mathematical ideas about logic problems to make choices about what the solution might, or might not, be
- Without help, and without waiting to be told, use what you have learned in one problem to say what you expect to find in other similar problems and draw some general conclusions about problems *of this type*
- Without any help, and without waiting to be told, explain how you know your answer is right even for the most complicated problems involving lots of information
- Without any help, and without waiting to be told, use mathematical ideas to decide whether your answer is correct, sensible and complete in every respect

Key Questions

Key questions in SPEAR Math colors at three levels of difficulty (K&G1, G2&3 and G4,5&6)

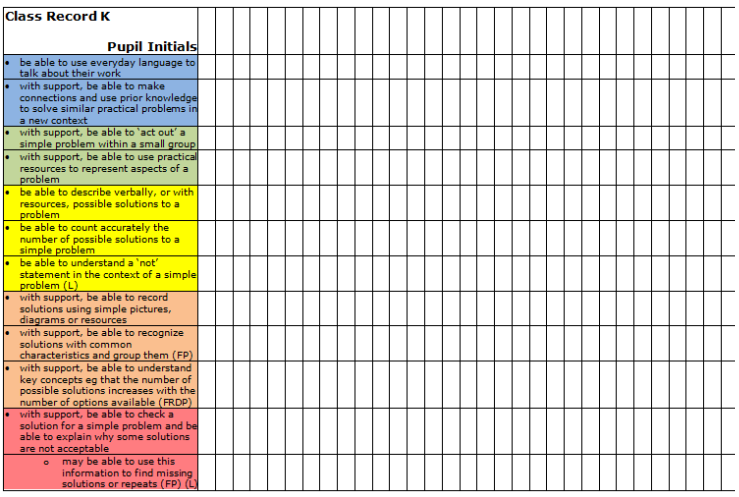
Specific Key Questions for each type of problem

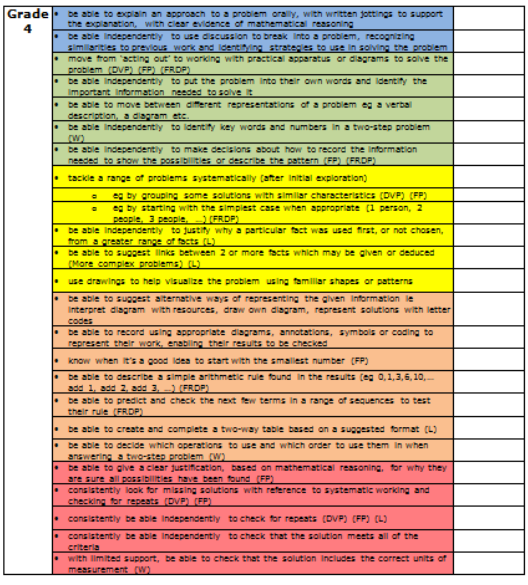




Next Step Records

Pupil progress records for all problem types for individual pupils and class records





Target Cards

Individual pupil Target Cards for Kindergarten to G6

Grade 1 Targets for Math Problem Solving	Date
With a little help, I can decide how I am going to start With a little help, I can decide on what's important to think about	
With a little help, I can tell an adult how I will use something I learned before to solve this problem	
I can say what I would like to use to help me solve the problem	
With a little help, I can act out the problem with other people	
With a little help, I can find all the answers that I can	
With help, I can decide how to solve the problem one step at a time	
I can remember that I can choose which clue to start with	
With a little help, I can explain why some clues go together to tell you something important	
I can explain to an adult what I did to try and solve the problem	
I can make drawings about the problem I am working on to show what I am doing	
I can draw pictures to show my answer to the problem	
I can put my answers in groups that go together if this is helpful	
I can talk to an adult about any patterns I have noticed	
With a little help, I can explain to an adult how I know that my answer is right	
With help, I can organize my answers and show how to find missing answers or answers I have got twice	
With a little help, I can explain to an adult how I know that there are no more answers to find	
With help, I can make sure I said what my answer is measured in	

Grade 5 Targets for Math Problem Solving	Date	Date	Date	Date	Date	Date
I can use pictures, writing, numbers and talk to clearly explain the mathematical ideas I am using to solve the problem						
I can think about questions to do with the problem and try and answer them using mathematical ideas						
I can think of my own ways to tackle a problem I have never seen before by remembering how I solved problems in the past, if this is helpful						
I can use my imagination to come up with new ways of trying to solve a problem						
I can highlight the words and numbers in a problem that I don't need to use in order to solve it						
I can make a good choice about the way to record all the answers I find that will be correct to do and clearest for others to understand						
I can use drawings, notes or symbols to help me and others to understand a problem better						
I can organise the important information into a list or table, as I decide, in order to see what might be missing						
I can explain my choices about where I started with the problem and put other clues into a sensible order of importance						
I can keep reading all the facts and looking for facts which go together to tell me something important and help me solve the problem						
I can choose one piece of information and see what happens when I use this information in the problem						
I can write down my answers in a clear and organised way using symbols that other people can understand						
I can make sense of a diagram or picture, allowing that I understand exactly what it is telling me, for example which answers are not allowed						
Without help, I can find a way to work that allows me to be clear about what I have tried and what I still need to do						
I can choose to only record what I need to record and I am able to explain my choices about this and, when I have found some results, say what else I expect to find using mathematical ideas to give reasons						
I can look carefully for patterns, describe any patterns I notice and then see if the next few numbers in the pattern are as I expected (say what I think the next number will be and test this)						
I can describe the pattern I have found in a way that other people can understand and use the pattern to say what another result would be, for example if I used a much bigger number						
Without help, I can choose the best way to organise my results (as I find them) and my answer(s) to the problem						
I can decide whether to add (+), subtract (-), multiply (x) or divide (÷) and in which order I need to do these to find the answer(s)						
I can use mathematical ideas and mathematical language to explain clearly how I know my answer(s) are correct						
I can use what I have learned in one problem to say what I expect to find in other problems before I do them						
Without help, I can explain how I know my answer is right even for difficult problems involving lots of information						
Without help, I can make sure I always say what units my answer is measured in						

Self Evaluation Sheets

Self Evaluation Sheets for each type of problem with or without a teacher's column.

Name	Date		
Problem Title		Pupil	Teacher
I read through the problem carefully and made sure that I understood what it was about			
I thought about similar problems I had seen in the past and decided whether I could use a similar approach			
I had a way to start exploring the problem			
I had a system for deciding on the information needed to describe the pattern			
I worked systematically, starting with the simplest case			
I organized the data I collected into a list or table to help identify any patterns that there were			
I chose an appropriate way of recording my ideas about what came next in the pattern			
I predicted what came next and tested this to see if I was right			
I had a way to track what had been included and what had not			
I had a way to find the general rule			
I used the general rule to say whether a number or shape would be in the sequence or not			
I checked my work to make sure it was accurate			
I decided if I had fully answered the question or solved the problem			

S	earch
P	lan
E	xplore
A	pply
R	eview

Name	Date
Problem Title	
I read through the problem carefully and made sure that I understood what it was about	
I thought about similar problems I had seen in the past and decided whether I could use a similar approach	
I had a way to start exploring the problem	
I identified the given facts and put them in order (I prioritized them)	
I looked for any relationships and patterns in the information given	
I worked systematically	
I had a way to use one piece of information at a time and see what effect it has, then to keep one thing fixed and test the other	
I chose an appropriate recording system	
I used the recording system to organize the information given in the problem	
I checked my work to make sure it was accurate	
I checked for any repeats and removed them	
I checked that the answer met all the criteria	
I decided if I had fully answered the question or solved the problem	

S	earch
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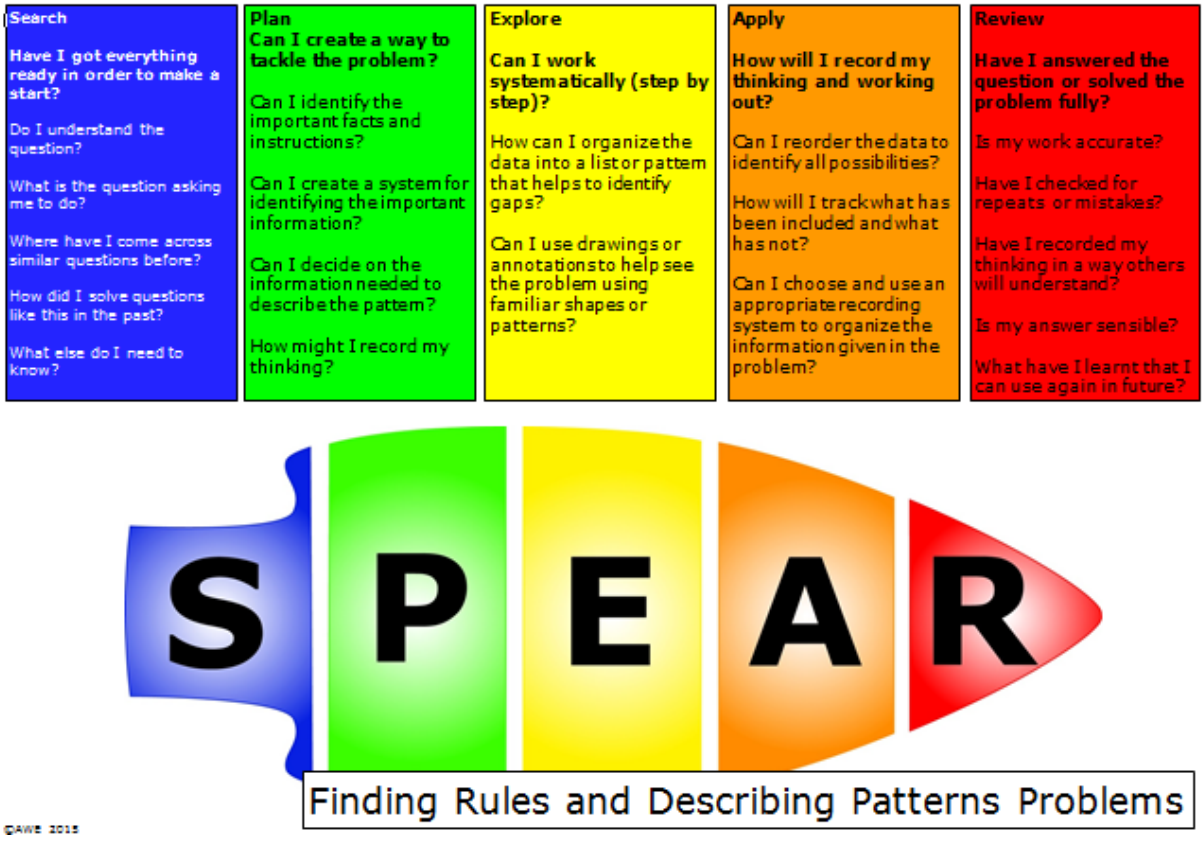
SPEAR Graphics

SPEAR graphic with key questions for each problem type:



Simplified SPEAR graphic with icons for younger children:

Teachers' Guides

Teachers' Guides for all problem types containing Statements of Attainment with examples of achieving each objective and suggested next teaching steps.

Grade 1 Teachers' Guide

Grade	Descriptor	Example	Next Teaching Steps
Grade 1	• with limited support, be able to find a starting point for solving a problem, identifying key facts and relevant information	Is able to choose from the (range of) resources available and decide how to use them to solve the problem Is able to solve problems involving addition, subtraction, multiplication and division in the context of numbers, measures or money	Create opportunities for children to use prior mathematical knowledge to solve problems eg you have $2, buy two items from the class shop to spend as much of your $2 as you can. Create opportunities for pupils to identify resources that are not immediately available
	• with limited support, be able to transfer knowledge and use it to solve problems	With limited support, is beginning to transfer mathematical knowledge and use it to solve problems eg find the difference between two ribbons that are 24 inches and 35 inches long; is able to recognize the connection between the number of wheels on three cars and the number of legs on three cows	Create opportunities for pupils to use classroom discussions to break into a problem, recognizing similarities to previous work and suggesting different approaches that could be used to solve it eg by asking questions such as 'have you done anything like this before?' and 'how could we find out?'
	• be able to use practical resources to accurately represent aspects of a problem	With support, is able to use apparatus, diagrams, role play etc. to represent and clarify a practical problem and so identify the mathematical knowledge needed to solve it	Create opportunities for children to identify the resources they require, including resources that are not immediately available eg by asking 'what would you like to use to help you?'
	• with some independence, be able to 'act out' a simple problem within a small group	Is beginning independently to be able to choose the mathematics needed to explore simple problems through role-play eg to make different towers using three blocks of different colors	Create opportunities for pupils to begin to move from 'acting out' to *only* working with practical apparatus or to using diagrams to solve the problem eg by asking questions such as 'could you find a way to draw your ideas?'
	• with limited support, be able to find examples that satisfy the rules of a simple problem (FP)	With limited support is able eg to find ways to get a total of four using 1 to 5 digit cards	Create opportunities for pupils to work with increasing independence on a range of problem contexts requiring them to find examples that satisfy the rules of the problem, sometimes by keeping one thing the same

Pupil Passports

Suggested collections of graded and ordered problems arranged termly for K to G5 classes. Each passport contains a mixture of problem types of increasing levels of difficulty and includes a brief description of each problem and necessary prior knowledge:

Type					G4 Third Quarter			
D	L	P	R	W	Title	Description	Necessary Prior Knowledge	Date/ Comment
•					**D27 Fun with Pentominoes**	Find all 5-square pentominoes (and 4-square, and 3-square)	Work systematically. Understand reflective and rotational symmetry	
		•			**P54 Island Menu**	Combinations of items for dinner on a desert island	Be able to work systematically	
		•			**P48 Bus Routes**	Explore routes on a simple (6 node) network. Work out costs of routes	Add mentally small numbers Understand networks	
			•		**R34 Odd Adding**	Totals of series of odd numbers starting with 1	Recognize square numbers	
•					**D34 Five Rectangles**	How many squares can be arranged to make exactly five different rectangles?	Understand simple factors	
	•				**L34 Birthday Presents**	Use clues to work out who gave which present to whom	Be able to work systematically	
	•				**L36 How Old?**	Use clues to work out the ages of 4 people	Be able to work systematically	

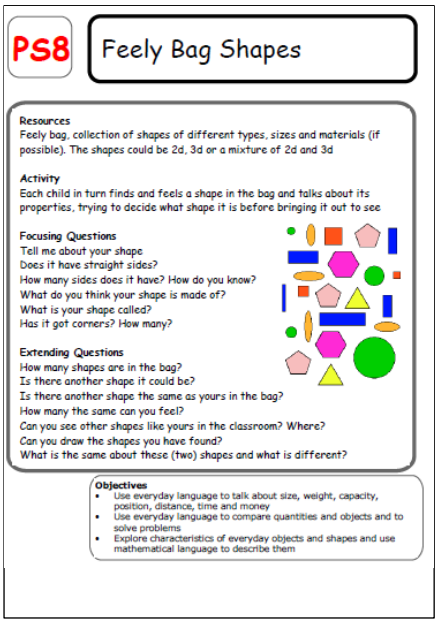
Problems

Over **550** problems covering Kindergarten to Grade 6 and all problem types in an easy to use **searchable** database. In addition, there are Preschool ideas for Math problem solving. New problems are being added all the time. Many problems now include ready-made resources such as empty tables, as well as extension and support materials. All problems include complete answers. Over 100 of the problems include simplified versions covering the same concepts.

Title	Category	Year Group	Description	Necessary Prior Knowledge	Key Learning Objectives
P1 Lollipops	Finding all Possibilities	1	Finds combinations of coins adding to 6p, 7p	Recognise coin values and understand the order of the values Add more than two numbers, 5 or less Count in ones and twos	Recognise simple patterns or relationships Generalise and predict Suggest extensions by asking 'what if??' or 'what could I try next?' Organise the recording of possibilities eg in an ordered list
P2 Down the path	Finding all Possibilities	1	Find pairs of numbers adding to 6	Addition of two numbers Counting on Pairs of numbers that make 6 Playing a track type game	Recognise simple patterns or relationships Generalise and predict Suggest extensions by asking 'what if??' or 'what could I try next?' Organise the recording of possibilities eg in an ordered list Begin to have a system for finding the possibilities eg start with the smallest number
P3 Four pin bowling	Finding all Possibilities	1	Add two numbers 1-4 to make 5, 6, 7	Addition and subtraction facts up to 10	Recognise simple patterns or relationships Organise the recording of possibilities eg in an ordered list
P4 Pick a pair	Finding all Possibilities	1	The sum of pairs of numbers chosen from 1, 2, 4 and 8	Addition and subtraction facts up to 10 Simple mental addition and subtraction	Recognise simple patterns or relationships Organise the recording of possibilities eg in an ordered list Begin to have a system for finding the possibilities eg start with the smallest numbers
P5 The wizard and the frogs	Finding all Possibilities	1	Ways of dividing 5 into two sets	Number Bonds to 5	Recognise simple patterns or relationships Generalise and predict Suggest extensions by asking 'what if??' or 'what could I try next?' Organise the recording of possibilities eg in an ordered list

Preschool Ideas

These activity sheets are intended to be used by adults to inform their support of pupils as they experience learning opportunities. Each sheet includes an activity, resources and focusing and extending questions, as well as key objectives:



PS8 Feely Bag Shapes

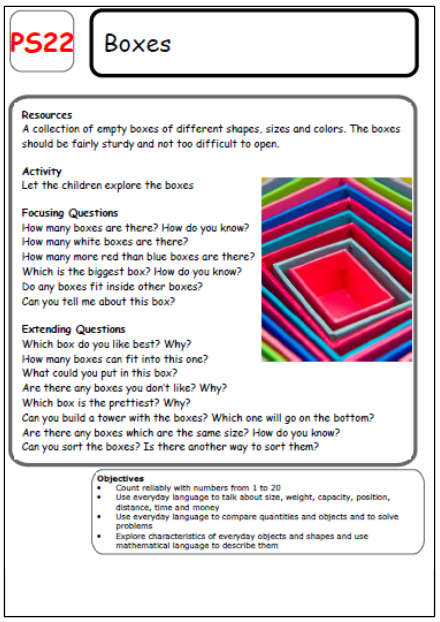
Resources
Feely bag, collection of shapes of different types, sizes and materials (if possible). The shapes could be 2d, 3d or a mixture of 2d and 3d

Activity
Each child in turn finds and feels a shape in the bag and talks about its properties, trying to decide what shape it is before bringing it out to see

Focusing Questions
Tell me about your shape
Does it have straight sides?
How many sides does it have? How do you know?
What do you think your shape is made of?
What is your shape called?
Has it got corners? How many?

Extending Questions
How many shapes are in the bag?
Is there another shape it could be?
Is there another shape the same as yours in the bag?
How many the same can you feel?
Can you see other shapes like yours in the classroom? Where?
Can you draw the shapes you have found?
What is the same about these (two) shapes and what is different?

Objectives
- Use everyday language to talk about size, weight, capacity, position, distance, time and money
- Use everyday language to compare quantities and objects and to solve problems
- Explore characteristics of everyday objects and shapes and use mathematical language to describe them



PS22 Boxes

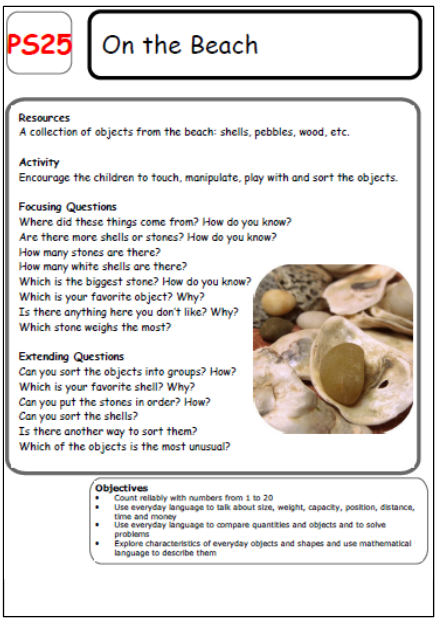
Resources
A collection of empty boxes of different shapes, sizes and colors. The boxes should be fairly sturdy and not too difficult to open.

Activity
Let the children explore the boxes

Focusing Questions
How many boxes are there? How do you know?
How many white boxes are there?
How many more red than blue boxes are there?
Which is the biggest box? How do you know?
Do any boxes fit inside other boxes?
Can you tell me about this box?

Extending Questions
Which box do you like best? Why?
How many boxes can fit into this one?
What could you put in this box?
Are there any boxes you don't like? Why?
Which box is the prettiest? Why?
Can you build a tower with the boxes? Which one will go on the bottom?
Are there any boxes which are the same size? How do you know?
Can you sort the boxes? Is there another way to sort them?

Objectives
- Count reliably with numbers from 1 to 20
- Use everyday language to talk about size, weight, capacity, position, distance, time and money
- Use everyday language to compare quantities and objects and to solve problems
- Explore characteristics of everyday objects and shapes and use mathematical language to describe them



PS25 On the Beach

Resources
A collection of objects from the beach: shells, pebbles, wood, etc.

Activity
Encourage the children to touch, manipulate, play with and sort the objects.

Focusing Questions
Where did these things come from? How do you know?
Are there more shells or stones? How do you know?
How many stones are there?
How many white shells are there?
Which is the biggest stone? How do you know?
Which is your favorite object? Why?
Is there anything here you don't like? Why?
Which stone weighs the most?

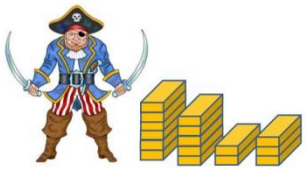
Extending Questions
Can you sort the objects into groups? How?
Which is your favorite shell? Why?
Can you put the stones in order? How?
Can you sort the shells?
Is there another way to sort them?
Which of the objects is the most unusual?

Objectives
- Count reliably with numbers from 1 to 20
- Use everyday language to talk about size, weight, capacity, position, distance, time and money
- Use everyday language to compare quantities and objects and to solve problems
- Explore characteristics of everyday objects and shapes and use mathematical language to describe them

Examples of Problems: Diagram and Visual Puzzles

D1 Gold Bars

Pete is a pirate.
His gold bars are in piles.
He can move one or more bars at a time.



He makes all the piles the same height.
He makes just two moves.
How does he do it?

Objectives
- Use drawings or resources to help visualise the problem
- Test ideas to see if they work
- Explain methods and reasoning
- Use a systematic approach to solve the problem

 Source: Mathematical Challenges for Able Pupils 7

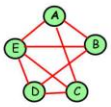
D15 Tony's Ice Cream

Tony is an ice cream man.

He visits 5 towns each day.

He gets bored visiting them in the same order every day.

Tony lives in town A, so he starts and finishes there every day



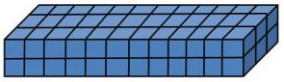
Using the map, can you list all the different possible routes so Tony can vary which way he travels each day?

Objectives
- Use drawings or annotations to help visualise the problem
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording

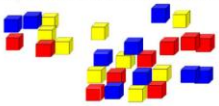
 Source: Stella Byrne (A Problem Shared)

D19 Seventy-two Cubes

Seventy-two cubes can be used to make a cuboid two cubes high, three cubes deep and 12 cubes long (2 x 3 x 12 = 72) like this:



How many other cuboids can be made using **exactly** 72 cubes?



How can you be sure that you have found them all? You might want to record your answers in a table.

Objectives
- Use drawings or resources to help visualise the problem
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording
- Try other possibilities to test the solution

 Source: NNS Supplement Y456

D25 House Numbers

On my street, the houses on one side of the street are numbered 1, 2, 3, 4 and so on.
The houses on the other side of the street are numbered backwards, so that the house with the largest number is opposite house number 1.

My house number is 12 and I live opposite house number 29.
How many houses are on my street?

If the houses are re-numbered so that one side of the street is the even numbers 2, 4, 6 and so on and the other side of the street is the odd numbers with the largest odd number opposite house number 2, which house number will be opposite mine?

Objectives
- Use drawings or annotations to help visualise the problem
- Choose and use an appropriate method of recording
- Use a systematic approach to solve the problem
- Identify the given information and represent it in another way

 Source: Stella Byrne (A Problem Shared)

D31 Garden Perimeter

You have been asked to design a garden with an area of 24 square metres. The shape of the garden is up to you. The garden will have to be fenced for security, so a shape with a shorter perimeter is better than one with a longer perimeter (as you will need to buy less fencing).

Using 1cm squared paper, explore the perimeters of shapes with an area of 24 square centimetres.

NOTE: We are using 1cm squared to represent a 'real life' area of 1 metre squared. In other words, we are using a scale of 1:100.

Start with simple shapes like rectangles. What about a square?
What about a circle?
What about more complex shapes?

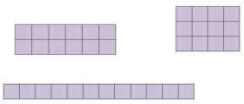
Which shape would be best?

Objectives
- Use drawings or annotations to help visualise the problem
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording
- Visualise 2d shapes

 Source: AJW

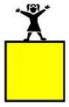
D34 Five Rectangles

With twelve squares you can make *exactly three* different *rectangles*.



Find out how many squares can be arranged to make *exactly five* different rectangles.

How does this link with factors?

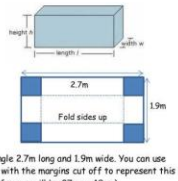


Objectives
- Use drawings or annotations to help visualise the problem
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording
- Try other possibilities to test the solution

 Source: NNS Supplement Y456

D38 Garden Pond

Your school wants a 'formal' rectangular pond. Of course, it's not really a rectangle, it's a cuboid! The pond will be 'raised' (it will sit on the ground and the water will be held in by walls).



The pond liner you have is a rectangle 2.7m long and 1.9m wide. You can use an A4 sheet of 1cm squared paper with the margins cut off to represent this size at a scale of 1:10 (the sheet of paper will be 27cm x 19cm).

Work out the biggest rectangular pond you can make using the liner available. The 'biggest' pond is the one that can hold the most water. The volume of a cuboid is given by:
l x b x h (which means 'length' x 'breadth' x 'height')

Objectives
- Use drawings or annotations to help visualise the problem
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording
- Try other possibilities to test the solution

 Source: AJW

D61 Ski Lift +

On a ski lift the chairs are equally spaced.
They are numbered in order from 1.

Kelly went skiing.

She got in chair 17 to go to the top of the slopes.

Exactly half way to the top, she passed chair 93 on its way down.

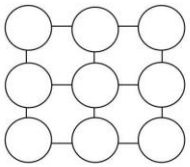
How many chairs are there on the ski lift?

Objectives
- Use drawings or resources to help visualise the problem
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording

 Source: AJW

D64 Odd Square

Put the numbers 1-9 in the circles so that the difference between each pair of joined numbers is odd:



A 'difference' is what you get when subtract the smaller number from the larger number

Note: there are lots of ways to do this!

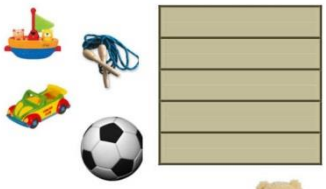
Objectives
- Use drawings or annotations to help visualise the problem
- Use a systematic approach to solve the problem

 Source: NNS Supplement Y123

Logic Problems

L1 Toys

Follow the clues to put the toys on the shelves:



The boat is on the middle shelf.
The skipping rope is on the top shelf.
The car is on the bottom shelf.
The ball is next to the boat.
The teddy is in between the boat and the car.

Objectives
- Recognise simple patterns or relationships, generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Use one piece of information at a time and see what effect it has
- Check that the answer meets all of the criteria

 Source: PNS Problem Solving DfES 1386-20054G

L5 Sally's Super Sandwich Shop

Sally runs a sandwich shop. She gets a very muddled order for lunchtime sandwiches from the office next door.
Can you sort it out using the clues?

How many of each sandwich must Sally make for the office?
Sandwiches can be brown bread or white bread.
They can be cheese or salad.

Order
We need 6 white bread sandwiches.
We need 2 white bread sandwiches with cheese.
We need 9 cheese sandwiches.
We need double the number of brown bread salad sandwiches as white bread salad sandwiches.

Objectives
- Recognise simple patterns or relationships, generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Use one piece of information at a time and see what effect it has
- Look for any relationships and patterns in the information given
- Check that the answer meets all of the criteria
- Solve a problem by identifying given facts and prioritising them
- Use recording to make sense of the information given and to find missing information

 Source: PNS Problem Solving DfES 1386-20054G

L15 Take it Easy

Take

Here are the rules:

Start with seven cubes or counters

Take turns

When it's your turn, you must take **one** or **two** cubes
The person who takes the **last** cube is the loser

Can you see how to **win?**
Remember: the person who takes the last cube is the loser

When you have worked out how to win every time, try making the person who takes the last cube the winner. How can you win now?

Objectives
- Recognise simple patterns or relationships, generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Use one piece of information at a time and see what effect it has
- Use recording to make sense of the information given and to find missing information

 Source: Unknown

L20 Rows of Coins

1. Take five coins: 1p, 2p, 5p, 10p, 20p.
Put them in a row using these clues. The total of the first three coins is 27p. The total of the last three coins is 31p.
The last coin is double the value of the first coin.

2. Take six coins: two 1p, two 2p and two 5p.
Put them in a row using these clues. Between the two 1p coins there is one coin. Between the two 2p coins there are two coins. Between the two 5p coins there are three coins.

What if you take two 10p coins as well as the six listed in question 2, and between them are four coins?

Objectives
- Use one piece of information at a time and see what effect it has
- Solve a problem by identifying given facts and prioritising them
- Look for any relationships and patterns in the information given
- Check that the answer meets all of the criteria

 Source: Mathematical Challenges for Able Pupils 26

L28 K x A

K x A = DF
F x A = BD
D x A = H
E x A = BE
J x A = DG
H x A = BK
B x A = A
G x A = DB
BC x A = AC
BD x A = AH
BB x A = AA
A x A = J

Each letter (in the equations on the left of the page) stands for a single digit.

Where there are two letters next to each other, this stands for a two-digit number.

Each letter stands for the **same** digit throughout all the equations.

Can you work out what digit each letter stands for?

Hint: You don't have to start at the top!

Objectives
- Recognise simple patterns or relationships, generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Look for any relationships and patterns in the information given
- Use recording to make sense of the information given and to find missing information
- Check that the answer meets all of the criteria

 Source: Unknown

L29 Nicknames

Dawn, Mark, Josh and Tina are friends. They each have a nickname. Their nicknames are Spider, Curly, Ace and Fudgy, but not in that order.

What is the nickname of each of the friends? Here are some clues to help you:

Josh plays tennis with Curly and goes swimming with Ace.
Tina has been on holiday with Curly but travels to school with Fudgy.
Spider, Curly and Dawn play in the football team.
Spider sometimes goes to tea with Josh.

Objectives
- Recognise simple patterns or relationships, generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Use one piece of information at a time and see what effect it has
- Check that the answer meets all of the criteria
- Solve a problem by identifying given facts and prioritising them
- Use recording to make sense of the information given and to find missing information

 Source: PNS Problem Solving DfES 1386-20054G

L39 Send in the Clowns

Six clowns stand in a line. Two have red noses, two have blue noses and two have green noses.

Use the clues to decide the colour of each clown's nose:
There is one nose between the two red noses
There are two noses between the two blue noses
There are three noses between the two green noses
(There are two possible answers that fit these clues)

Now find as many answers as you that match these clues:
The green noses are next to each other
There is a blue nose on one end of the line but the other blue nose is not on the other end of the line
The red noses are not next to each other

Objectives
- Solve a problem by identifying given facts and prioritising them
- Use one piece of information at a time and see what effect it has
- Look for any relationships and patterns in the information given
- Use recording to make sense of the information given and to find missing information
- Check that the answer meets all of the criteria

 Source: Stella Byrne (A Problem Shared)

L47 At The Zoo

At The Zoo

Three girls visit the zoo. Can you find their favourite animal, what lolly they had to eat and with whom they went?

Girl:	*Ellie, Emily, Jessica*
Animal:	*Elephant, Zebra, Giraffe*
Lolly:	*Grunge, Plooper, Zinger*
With:	*Aunt, Grandma, Mum*

1. Ellie went to the zoo with her Grandma, but did not have a Zinger.

2. Mum went with the girl whose favourite animal was the elephant.

3. The girl whose favourite was the giraffe did not have a Plooper.

4. The Aunt bought a Grunge for the child she took to the zoo, this was not Jessica.

Objectives
- Solve a problem by identifying given facts and prioritising them
- Use one piece of information at a time and see what effect it has
- Look for any relationships and patterns in the information given
- Use recording to make sense of the information given and to find missing information
- Check that the answer meets all of the criteria

 Source: Unknown

L57 Lunchbox Riddle

Amy, Billy, Chloe, Danielle and Emma each own a lunchbox. The lunchboxes are in a straight line in the dining hall. Each lunchbox is a different colour and contains a sandwich, a drink and a piece of fruit. Each child has a different sandwich, a different drink and a different piece of fruit. **Who has a pear for lunch?**

Facts:
Amy has a red lunchbox.
Danielle has a banana for lunch.
Billy only drinks squash.
Chloe only eats jam sandwiches.
The first lunchbox belongs to Emma.
The lunchbox in the middle contains milk.
Emma's lunchbox is next to the blue one.
The green lunchbox's owner drinks lemonade.
The person who eats tuna sandwiches has a peach.
The owner of the yellow lunchbox has a ham sandwich.
The green lunchbox is on the left of the white lunchbox.
The lunchbox with the egg sandwich is next to the one with the orange.
The apple is in the lunchbox next to the one with the ham sandwich.
The owner of the cheese sandwich has cola to drink.
The water-drinker's box is next to the egg-eater's box.

Objectives
- Recognise simple patterns or relationships, generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Solve a problem by identifying given facts and prioritising them
- Look for any relationships and patterns in the information given
- Use one piece of information at a time and see what effect it has
- Use recording to make sense of the information given and to find missing information
- Check that the answer meets all of the criteria

 Source: Stella Byrne (A Problem Shared)

Finding All Possibilities Problems

P1 Lollipops

Jodie bought a lollipop. It cost 6p.

She paid for it exactly. Which coins did she use?
There are five different ways to do it. How many of these ways can you find?

How many ways would there be if the lollipop cost 7p?

Objectives
- Recognise simple patterns or relationships
- Generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Organise the recording of possibilities eg in an ordered list

 Source: PNS Problem Solving DfES 0545-2004G

P14 Buttons

Lisa has 7 buttons. She puts them into 3 boxes:

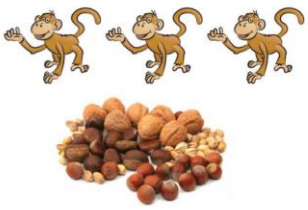
How many different ways can Lisa sort 7 buttons into 3 identical boxes?
Show your thinking.

Objectives
- Recognise simple patterns or relationships
- Generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Organise the recording of possibilities eg in an ordered list or table
- Have a system for finding the possibilities eg start with the smallest number, know when all possibilities are found, check for repeats of possibilities

 Source: Stella Byrne (A Problem Shared)

P25 Three Monkeys

Three monkeys ate a total of 25 nuts.
Each of them ate a different odd number of nuts.



How many nuts did each of the monkeys eat?
Find as many different ways to do it as you can.

Objectives
- Recognise simple patterns or relationships
- Generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Organise the recording of possibilities eg in an ordered list or table
- Have a system for finding the possibilities eg start with the smallest number, know when all possibilities are found, check for repeats of possibilities

 Source: Mathematical Challenges for Able Pupils 31

P43 Lining Up

In how many different orders can a group of children line up at the door?

One person? Two people?

What about three people? Four people?
Put this information into a table and look for a pattern?

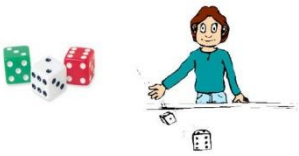
In how many orders can 10 people line up?
Can you find the general rule: How many orders for N people?

Objectives
- Use drawings or annotations to help visualise the problem
- Recognise patterns or relationships
- Organise the recording of possibilities eg in an ordered list
- Have a system for finding the possibilities eg start with the smallest number, know when all possibilities are found, check for repeats of possibilities

 Source: AJW

P52 Three Dice

Sigmund rolls three dice



He adds up all three numbers.

How many different totals could he get?
Try to work systematically.
Show your thinking.

Objectives
- Recognise patterns or relationships
- Organise the recording of possibilities eg in an ordered list
- Have a system for finding the possibilities eg start with the smallest number, know when all possibilities are found, check for repeats of possibilities

 Source: Stella Byrne (A Problem Shared)

P54 Island Menu

Oh dear, you are shipwrecked on a desert island. You have had a look round and managed to collect the following foods for dinner:

Bread
Fish
Bananas
Yams
Berries
Seaweed

*For this problem, assume that the **order** you choose the items is **important**, so a choice of 'fish' first and 'bread' second is **different** to choosing 'bread' first and 'fish' second.*

How many different ways can you choose:

a) One item for dinner?
b) Two different items for dinner?
c) Three different items for dinner?

What if the order **isn't** important?
How many different ways can you choose one, or two, or three items now? (Note: this is a *much* harder problem!)

Objectives
- Use drawings or annotations to help visualise the problem
- Organise the recording of possibilities eg in an ordered list
- Have a system for finding the possibilities eg start with the smallest number, know when all possibilities are found, check for repeats of possibilities

 Source: AJW

P67 Half Time Score

The football match ended in a draw (the score could have been 0-0 or 1-1 or 2-2 or...)

What might the score have been at half time?
Start with a full time score of 0-0.
Work systematically.
Show your thinking.

Objectives
- Recognise patterns or relationships
- Generalise and predict
- Suggest extensions by asking 'what if...?' or 'what could I try next?'
- Organise the recording of possibilities eg in an ordered list
- Have a system for finding the possibilities, know when all possibilities are found, check for repeats of possibilities

 Source: Stella Byrne (A Problem Shared)

P104 Tracksuits +

You have 3 new tracksuits *with matching socks*:

One morning you get dressed in the dark. You reach into your clothes cupboard and take out a top, a pair of trousers and a pair of socks without looking.

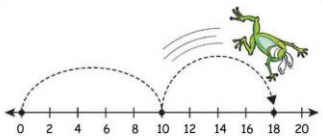
How many different possible outfits are there?

If you bought a yellow tracksuit with matching socks, how many **more** possibilities will there be?
Show your thinking.

Objectives
- Have a system for finding the possibilities eg start with the smallest number, know when all possibilities are found, check for repeats of possibilities
- Organise the recording of possibilities eg in an ordered list or table
- Use a method of tracking what has been included and what has not

 Source: AJW

P144 Three Hops to 20



Freddie the Frog takes 3 hops to get from 0 to 20

One way he does it is: 10 + 8 + 2 = 20

How many ways can you find for Freddie to take 3 hops to get from 0 to 20?

Write each one down as a number sentence.

Objectives
- Organise the recording of possibilities eg in an ordered list
- Use a systematic approach to solve the problem
- Recognise simple patterns and relationships, generalise and predict

 Source: NNS Supplement Y123

Finding Rules and Describing Patterns Problems

R2 Teddy's Birthday Candles

Teddy is 7 years old today.

How many birthday candles has he blown out since he was born?

Objectives
- Describe and extend simple number sequences
- Recognise simple patterns and relationships, generalise and predict
- Solve and explain their solution to a given problem

 Source: PNS Problem Solving DfES 1387-2005G

R24 The 49th Counter

A line of counters is set out in a pattern: two white, four blue, two white, four blue and so on.

What colour is the 49th counter? How do you know?

What colour is the 49th counter in each of these patterns? How do you know?

Objectives
- Describe a rule of a pattern or relationship in words or symbols
- Decide on the information you need to continue the pattern
- Use drawings or annotations to help visualise the problem

 Source: NNS Supplement Y4S6

R27 Sequence of Shapes

Make these shapes with plastic cubes:

Can you see how the pattern develops?
Make a table of your results:

Shape Number	1	2	3	4	5	6
Number of Cubes	1	4	7			

How many cubes will there be in Shape 10?
How many cubes will there be in Shape 100?

Find a general rule for how many cubes there are in 'shape n' where 'n' stands for 'any shape you choose'

Objectives
- Describe a rule of a pattern or relationship in words or symbols
- Decide on the information you need to continue the pattern
- Predict the next few terms of a sequence to test the rule

 Source: PNS Problem Solving DfES 1387-2005G

R30 Tennis Tournament

A number of people take part in a tennis tournament.

In every round the winners of each game progress to the next round, the losers are out. If there is an odd number of people in a round, one player has a 'bye' to the next round.

How many games of tennis will be played in the tournament?
Look at tournaments with different numbers of players

Can you write a general rule to work out the number of games played for any number of players starting a tournament?

Objectives
- Describe a rule of a pattern or relationship in words or symbols
- Decide on the information you need to continue the pattern
- Predict the next few terms of a sequence to test the rule
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording

 Source: Stella Byrne (A Problem Shared)

R36 Handshakes

Everyone in a group shakes hand with everyone else once only.

How many handshakes is that all together?

Try and work systematically, starting with the easiest case. You could record your findings in a table:

No. of People in group	1	2	3	4	5	6
No. of Handshakes	0	1				

Can you find a general rule for the number of handshakes?

Objectives
- Describe a rule of a pattern or relationship in words or symbols
- Decide on the information you need to continue the pattern
- Predict the next few terms of a sequence to test the rule
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording

 Source: Stella Byrne (A Problem Shared)

R44 Square & Triangular Numbers

Test this statement and decide if it is true:

'Any square number is the sum of two consecutive triangular numbers'

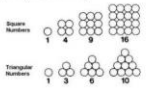
For example:

4 = 1 + 3 — 4 is a square number
1 and 3 are consecutive triangular numbers

Find other examples that match the statement.

Is the statement always true? Explain your thinking.

Square Numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 etc.
Triangular Numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45 etc.



Objectives
- Recognise and explain patterns or relationships, generalise and predict
- Describe a rule of a pattern or relationship in words or symbols
- Use a systematic approach to solve the problem

 Source: NNS Supplement Y4S6

R46 Unichains

You can make a chain by adding together any pair of digits and writing only the units digit total in the next link. For example, if the chain starts with 1 and 8:

1 + 8 = 9
8 + 9 = 17 (so write '7')
9 + 7 = 16 (so write '6')
7 + 6 = 13 (so write '3')
and so on!

Try starting with different pairs of numbers. Does the chain form a loop?
Try other pairs of digits. Note that the order makes a difference (starting with '8, 1' creates a different chain to starting with '1, 8')

Objectives
- Recognise and explain patterns or relationships, generalise and predict
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording

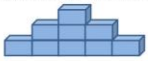
 Source: Math-Matrix 91/92 (Casio)

R86 Olympic Medals

At the Olympics, the winners stand on boxes to get their medals:

For three medal winners, four boxes are needed:

Four five medal winners, you need this arrangement of boxes:



How many boxes is this?

How many boxes will you need for 7 people to get medals? What about 9 people?

Can you find the general rule: How many boxes for any odd number of winners?

Objectives
- Describe a rule of a pattern or relationship in words or symbols
- Decide on the information you need to continue the pattern
- Predict the next few terms of a sequence to test the rule
- Use a systematic approach to solve the problem
- Choose and use an appropriate method of recording

 Source: 5-14 Maths support pack 'I Can Solve Problems' (Glasgow)

R88 Socks

Hamish has a pile of socks:

He decides to sort them into pairs:

He has one sock left over.

How many socks could Hamish have altogether?

He finds another sock to complete the final pair. How many socks could Hamish have altogether now?

Objectives
- Recognise and explain patterns or relationships, generalise and predict
- Describe a rule of a pattern or relationship in words or symbols
- Use a systematic approach to solve the problem

 Source: NNS Supplement Y123

Word Problems

There are currently **125** Y1-Y6 Word Problems in the SPEAR Math Problem Bank:

W2 Stickers

The twins collected some animal stickers. They each had the same total number. They stuck some in their sticker books and kept some loose for swaps.

Winston had 3 full sheets and 4 loose stickers. Wendy had 2 full sheets and 12 loose stickers.

Every full sheet has the same number of stickers. **How many stickers are there in a full sheet?**

Objectives
- Choose and use appropriate number operations and appropriate ways of calculating to solve problems
- Use a systematic approach to solve the problem
- Check that the answer meets all of the criteria

 Source: Mathematical Challenges for Able Pupils 42

W22 1, 2, 3 and 4 make...

Use **only** the digits 1, 2, 3, and 4 (one of each).

You can also use any operations you like: + - x ÷ as many times as you like

Here is a calculation that totals 1: 2 + 3 - 4 x 1 = 1

Here is a calculation that totals 40: 43 - 2 - 1 = 40

Now make up calculations that total 2, 3, 4, 5, 6, 7 etc.

Can you make each number from 1 to 40?

Create a way to work systematically.

Objectives
- Choose and use appropriate number operations and appropriate ways of calculating to solve problems
- Use a systematic approach to solve the problem
- Use all four operations to solve word problems involving numbers in 'real life'

 Source: NNS Supplement Y4S6

W25 Sandcastles

Lisa went on holiday. In 5 days she made 80 sandcastles.

Each day she made 4 fewer castles than the day before.
How many castles did she make each day?

Lisa went on making 4 fewer castles each day.
How many castles did she make altogether?

Objectives
- Choose and use appropriate number operations and appropriate ways of calculating to solve problems
- Recognise simple patterns or relationships, generalise and predict
- Use a systematic approach to solve the problem
- Check that the answer meets all of the criteria

 Source: Mathematical Challenges for Able Pupils 45

W38 Reading Rate

1. I started to read a book on Thursday. On Friday, I read 10 more pages than I read on Thursday and I got to page 60. **How many pages did I read on Thursday?**

2. I started to read another book on Monday. On Tuesday, I read 10 more pages than on Monday and I got to page 46. **How many pages did I read on Monday?**

3. I started a third book on Wednesday. Each day, I read 5 more pages than the day before. On Friday, I got to page 60. **How many pages did I read on Wednesday?**

Objectives
- Choose and use appropriate number operations and appropriate ways of calculating to solve problems
- Use a systematic approach to solve the problem
- Check that the answer meets all of the criteria

 Source: NNS Supplement Y4S6

W46 What makes 1?

Find as many different ways as you can to complete the following equation:

Organise your answers into groups of similar types

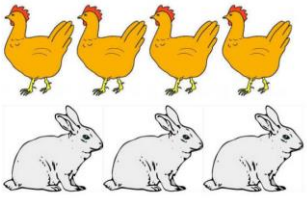
Answers using whole numbers	Answers using decimals	Answers using negative numbers	Answers using ...?
1+0+0=1			

Objectives
- Organise the recording of possibilities eg in an ordered list
- Begin to have a system for finding the possibilities eg start with the smallest number
- Choose and use an appropriate method of recording
- Use a systematic approach to solve the problem
- Check that the answer meets all of the criteria

 Source: NNS Supplement Y4S6

W48 Hens and Rabbits

A farmer has hens and rabbits. These animals have 50 heads and 140 feet altogether. How many hens are there and how many rabbits?



Objectives
- Recognise simple patterns or relationships, generalise and predict
- Choose and use an appropriate method of recording
- Use a systematic approach to solve the problem
- Check that the answer meets all of the criteria

 Source: Unknown

W53 Coins on the Table

Anna put some 10p coins on the table. One half of them were tails up.

Anna turned over two of the coins, and then one third of them were tails up.

How many coins did Anna put on the table?

Objectives
- Choose and use appropriate number operations and appropriate ways of calculating to solve problems
- Use a systematic approach to solve the problem
- Check that the answer meets all of the criteria

 Source: Mathematical Challenges for Able Pupils 69

W79 Lining Up Takes Time!

A group of 10 children can line up at the door in 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = **3628800 ways!**

If they could change position every second, how long would it take them to make all the different orders that are possible?
How many seconds? How many minutes is that? How many hours? How many days?
When you have found out how long it would take for 10 people to make all the different orders, work out how long it would take for 11 people to do the same.

Objectives
- Choose and use appropriate number operations and appropriate ways of calculating to solve problems
- Use a systematic approach to solve the problem
- Check that the answer meets all of the criteria

 Source: AJW

W97 Motorbikes and Limos

Motorbikes have two wheels. Limos have six wheels.

Ian saw a collection of motorbikes and limousines.
He counted 18 wheels altogether.

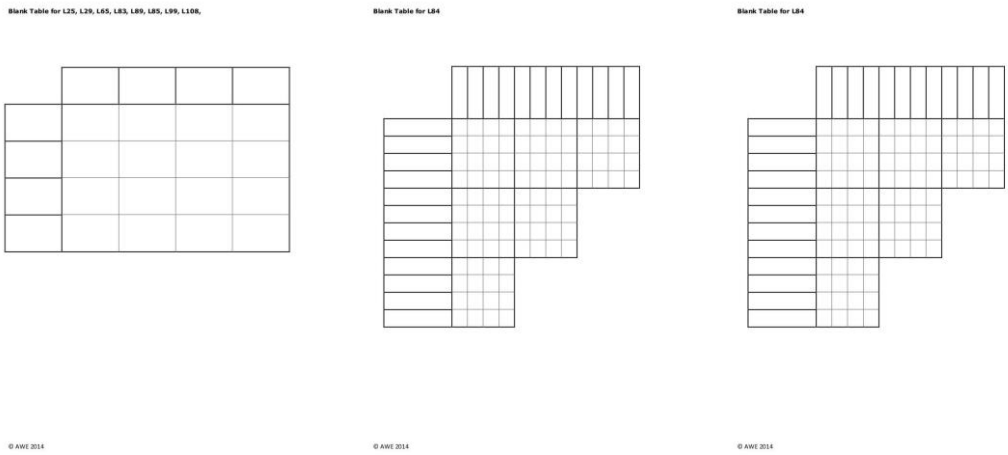
How many motorbikes were there and how many limousines?

Find as many different answers as you can.

Objectives
- Choose and use appropriate number operations and appropriate ways of calculating to solve problems
- Use a systematic approach to solve the problem
- Check that the answer meets all of the criteria

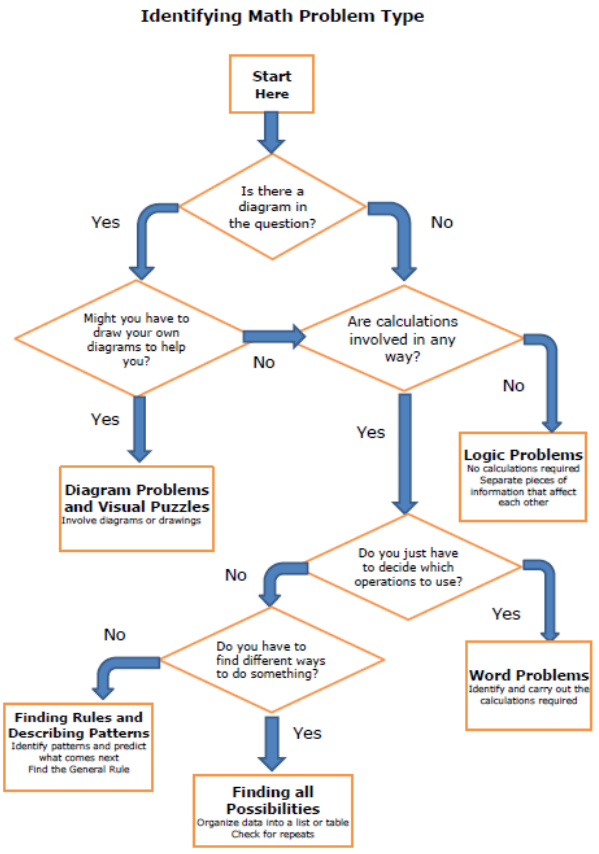
 Source: 5-14 Maths support pack "I Can Solve Problems" (Glasgow)

Other Resources included in SPEAR Math



A range of blank two way tables for use with Logic Problems

How to identify the type of problem



Coming Soon

- Core Materials for Grades 7 and 8
- Problems for Grades 7 and 8

Do you want to help develop SPEAR Math?

Get in touch with us and tell us what your pupils need. We are always looking for Development Partners to work with us to improve our resources.